

AERODYNAMIC SYMBOLS

1. GENERAL

- m mass
 t time
 V resultant linear velocity
 Ω resultant angular velocity
 ρ density, or relative density
 ν kinematic coefficient of viscosity
 R Reynolds number, $R = lV/\nu$ (where l is a suitable linear dimension), to be expressed as a numerical coefficient $\times 10^6$

Normal temperature and pressure for aeronautical work are 15°C . and 760 mm.

For air under these ($\rho = 0.002378$ slug/cu. ft. conditions ($\nu = 1.59 \times 10^{-4}$ sq. ft./sec.

The slug is taken to be 32.2 lb.-mass

- α angle of incidence
 ϵ angle of downwash
 S area
 c chord
 s semi-span
 A aspect ratio, $A = 4s^2/S$
 L lift, with coefficient $k_L = L/SpV^2$
 D drag, with coefficient $k_D = D/SpV^2$
 γ gliding angle, $\tan \gamma = D/L$
 L rolling moment, with coefficient $k_r = L/SpV$
 M pitching moment, with coefficient $k_m = M/SpV$
 N yawing moment, with coefficient $k_n = N/SpV$

2. AIRSCREWS

- n revolutions per second
 D diameter
 J V/nD
 P power
 T thrust, with coefficient $k_t = T/\rho n^2 D^4$
 Q torque, with coefficient $k_q = Q/\rho n^2 D^5$
 η efficiency, $\eta = TV/P = Jk_t/2\pi k_q$

DRAG OF FLAGS.

By R. A. FAIRTHORNE.

Communicated by the Director of Scientific Research, Air Ministry.

ARC Reports and Memoranda No. 1345.
 (Ae. 477.)

May, 1930.

Summary.—Introductory. (Purpose of investigation).—Figures were requested by the Airworthiness Department for the drag of banners towed by aircraft.

Range of investigation.—Measurements were made of the drag of rectangular flags of fineness ratio 0.5 to 4.0, and of triangular flags of fineness ratio 0.25 to 1.0. Two rectangular flags of materials of different densities were also tested.

Conclusions.—Triangular and rectangular flags of the same material, span and area seem to have the same drag. For flags weighing 0.016 lb./ft.² and 4 ft. span, the value of the drag coefficient k_D , based on unit projected area, ranged from 0.1 at fineness ratio 0.25 to 0.02 at fineness ratio 2. The part of the total drag not due to skin friction varies directly with the density of the material. An empirical relation between the drag and fineness ratio is given.

Owing to the use of banners towed by aircraft for advertising purposes, values of the drag of such flags were required by the Airworthiness Department.

Measurements of drag were therefore made in the wind tunnels, R.A.E., in November 1929, and January 1930. The flags tested were:—

- (1) An almost square flag, span $22\frac{1}{2}$ inches, fineness ratio 0.97, made of material weighing 0.032 lb./ft.². $\approx 156 \text{ g./m}^2$
- (2) Rectangular flags, fineness ratio from 2.0 to 0.5, of lawn weighing 0.016 lb./ft.². $\approx 78 \text{ g./m}^2$
- (3) Pennants, in the form of isosceles triangles with the base as leading edge, of fineness ratio 1.0 to 0.25, and the same material as (2). $\approx 78 \text{ g./m}^2$
- (4) A silk banner, 9 ft. 6 in. square, with 13 per cent. of the surface formed of lettering of muslin, the silk weighing 0.0075 lb./ft.². $\approx 37 \text{ g./m}^2$

Apparatus and method.—In case (1) the drag was measured by fixing the flag to the balance spindle. The forces were found to be periodic, and in subsequent tests, to avoid damage to the tunnel balance, the flags were attached to a spring balance by a wire running

over a pulley, the wire being fixed by silk cords to a batten in the leading edge.

In case (4), where the span was greater than the width of the tunnel, the leading edge was attached to a wire ring, 3 ft. in diameter, thus forming a cylinder slit down a generator. As a check on this method the ring was opened out to the form of a semicircle, giving a gap of 5 ft. 6 in. between the corners of the leading edge. No appreciable change in drag was observed.

The speed range was 60 to 80 ft./sec. in test (1), 60 to 100 ft./sec. in test (2), and 40 to 80 ft./sec. in tests (3) and (4).

The balance pointer oscillated at the frequency of the flag flutter, the amplitude of oscillation being especially large with the rectangular flags of test (2), for which the readings varied 30 per cent. from the mean at fineness ratio 2.0, and wind speed 100 ft./sec. The triangular flags were steadier, giving a maximum variation of 6 per cent. from the mean at fineness ratio 0.25, wind speed 80 ft./sec., while the flag of test (4) varied 4 per cent. from the mean at fineness ratio 1.0 and wind speed 80 ft./sec. The amplitude of the oscillation decreased as the fineness ratio increased.

The length of tow, i.e. the distance between the leading edge and pulley, was varied for the rectangular flag of fineness ratio 0.5, and was found to be of influence in that the shorter the tow, the greater was the drag. The variation was not large enough to affect the results, the length of tow being in all cases about 30 inches.

Results.

(a) *Drag*.—The total drag coefficients, drag at 100 ft.-sec. weight, and fineness ratio of the flags are given in Table I, and the drag coefficient is plotted against fineness ratio in Fig. 1. The drag values given are the mean over the range of speed tested. Within the accuracy of the experiment no systematic variation with speed was shown.

The drag coefficient of the triangular and rectangular flags of lawn was found to vary from 0.095 at fineness ratio 0.25, to 0.020 at fineness ratio 2.0, the fineness ratio being defined as the maximum projected area divided by the square of the span, and the drag coefficients being calculated for unit value of the maximum projected area.

The drag coefficient of the silk flag of fineness ratio 1.0 was 0.016, and that of the heavy flag weighing 0.032 lb./ft.² of fineness ratio 0.97 was 0.11.

After measurement of the drag of the rectangular flag of fineness ratio 0.48, the flag was slit down the centre to form two flags in parallel, each of fineness ratio 0.96. The process was repeated up to the formation of eight flags in parallel, each of fineness ratio 3.84. In each case the drag was lowered by subdivision, and presumably tends to the skin friction as limit. The drag and drag coefficients are given in Table II.

(b) *Frequency*.—An attempt was made to measure with a stroboscope the frequency of flutter of the triangular flags. There were several modes of vibration, but the frequency of the main progressive wave from leading edge to trailing edge could be measured approximately, and was found to vary from 21 per sec. at fineness ratio 1.0 to 38 per sec. at fineness ratio 0.25, both at a wind speed of 60 ft./sec. Values of the frequency at different speeds and fineness ratios are given in Table III.

Conclusions.—The fineness ratio can be regarded as a parameter fixing the shape of a flag of given geometrical type. For flapping flags of given shape, consideration of dynamical similarity shows that the drag coefficient at a given Reynolds number varies with the ratio of the effective density of the flag to the air density. This density ratio may be expressed by the factor $\frac{w}{s \rho g}$, where w is the

weight per unit area of the flag, s the span, ρ the volume density of the air, and g the acceleration due to gravity. The measured drag coefficients were practically independent of Reynolds number, and therefore for all flags of given geometrical type the drag coefficients may be taken to be the sum of two functions, the first of which is a function of the two variables, fineness ratio and the density ratio, and the other a function of the viscous forces, i.e. the skin friction.

The drag coefficients less skin friction, divided by the density ratio are given in Tables I and II. The value of the skin friction coefficients has been taken as 0.012, a figure suggested by tests carried out by the Göttingen aerodynamical laboratory on fabric in the form of a rectangle of fineness ratio 0.5, measured at about the same Reynolds number as the present tests.*

In Fig. 2 the drag has been plotted in this form against fineness ratio. The values approximate fairly closely to the empirical curve $y = 0.39 F^{-1.25}$, where y is $\frac{k_D - 0.012}{\text{density ratio}}$. The values for the triangular and rectangular flags lie approximately on a common curve.

In a subsequent test a rectangular streamer, 47.5 in. long and 2.95 in. wide, weighing 0.037 lb. per sq. ft., gave a mean value of 1.85 lb. for the drag at 100 ft./sec. when measured over a speed range of 60 to 100 ft./sec. The value calculated from the above formula is 1.87 lb., which is in good agreement with the measured values.

The general accuracy of the measurements is low, but it may be concluded that the quantity y is approximately constant for all flags, rectangular or triangular, with the same fineness ratio and the same aerodynamic roughness.

* Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen, Band I, Teil IV, "Untersuchungen über der Reibungswiderstand von stoffbespannten Flächen."

TABLE I.
Drag of Flags of Different Shape, Density and Fineness Ratio.

Rectangular.	2.0	1.5	1.0	0.75	0.5	0.48	0.25	1.0	0.97
Fineness ratio	0.016	0.016	0.016	—	—	0.016	—	0.0075	0.032
Weight per sq. ft. w	4	4	4	—	—	4	—	9.5	1.875
Span s (ft.)	0.0523	0.0523	0.0523	—	—	0.0523	—	0.0103	0.223
Density ratio $\sigma = \frac{w}{spg}$	15.0	14.3	12.4	—	—	10.4	—	34.0	9.0
Drag at 100 ft./sec. (lb.)	0.020	0.025	0.033	—	—	0.057	—	0.016	0.11
k_D	0.153	0.248	0.401	—	—	0.860	—	0.388	0.439
$k_D - 0.012$	—	—	—	—	—	—	—	—	—
σ	—	—	—	—	—	—	—	—	—
Triangular.	—	—	—	—	—	—	—	—	—
Weight per sq. ft. w	—	—	—	—	—	—	—	—	—
Span s (ft.)	—	—	—	—	—	—	—	—	—
Density ratio σ	—	—	—	—	—	—	—	—	—
Drag at 100 ft./sec. (lb.)	—	—	—	—	—	—	—	—	—
k_D	—	—	—	—	—	—	—	—	—
$k_D - 0.012$	—	—	—	—	—	—	—	—	—
σ	—	—	—	—	—	—	—	—	—

TABLE II.

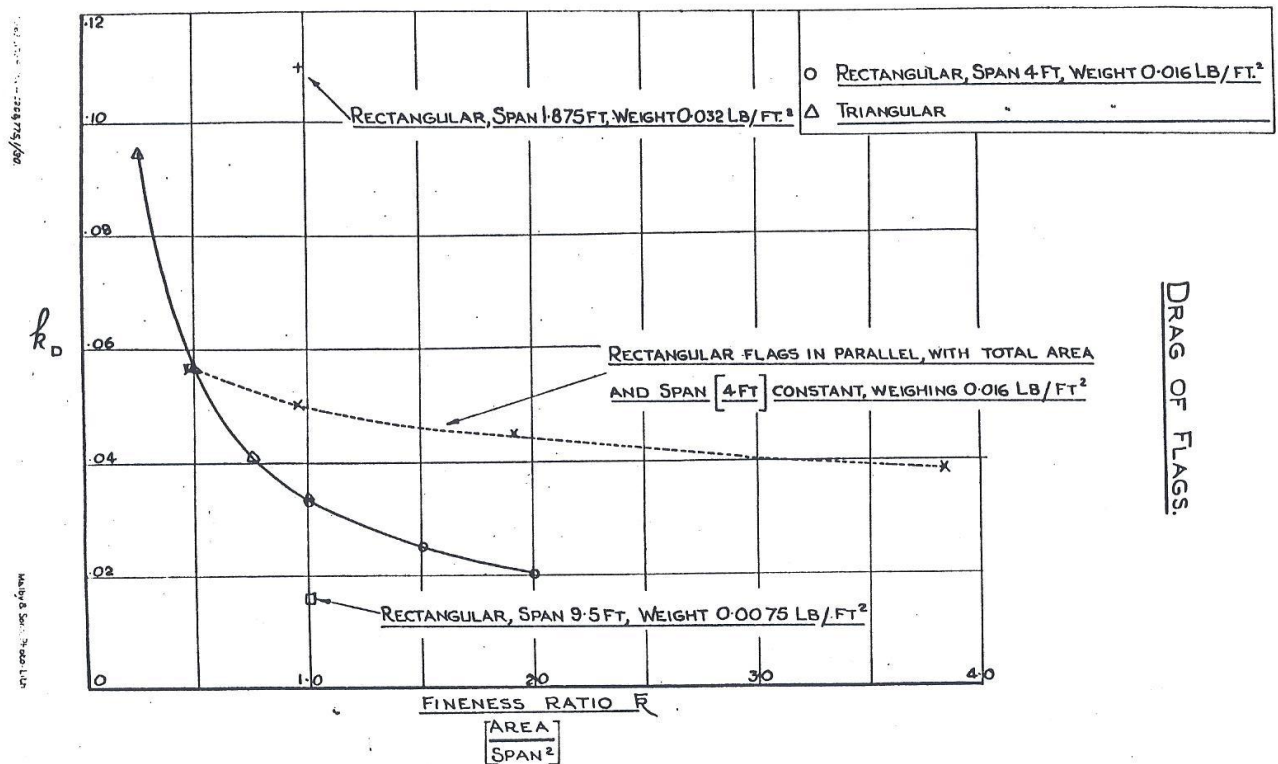
Drag of Rectangular Flags in Parallel with Constant Total Area and Span.

Number of Flags.	1.	2.	4.	8.
Fineness ratio	0.48	0.96	1.92	3.84
Density ratio σ	0.052	0.105	0.209	0.418
Drag at 100 ft./sec. (lb.) ..	10.4	9.4	8.1	7.0
k_D	0.057	0.051	0.045	0.039
$k_D - 0.012$	0.860	0.373	0.158	0.065
σ	—	—	—	—

TABLE III.

Frequency of Flutter of Triangular Flags of Span 4 ft.

Fineness Ratio.	1.0.	0.75.	0.5.	0.25.	Wind speed. ft./sec.
Frequency per sec. ..	18.3	19.5	20.3	27.7	40
" ..	20.8	22.5	27.0	38.4	60
" ..	—	28.4	35.0	—	80



R&M.1345.

FIG. 1.

DRAG OF FLAGS.

DRAG OF FLAGS.

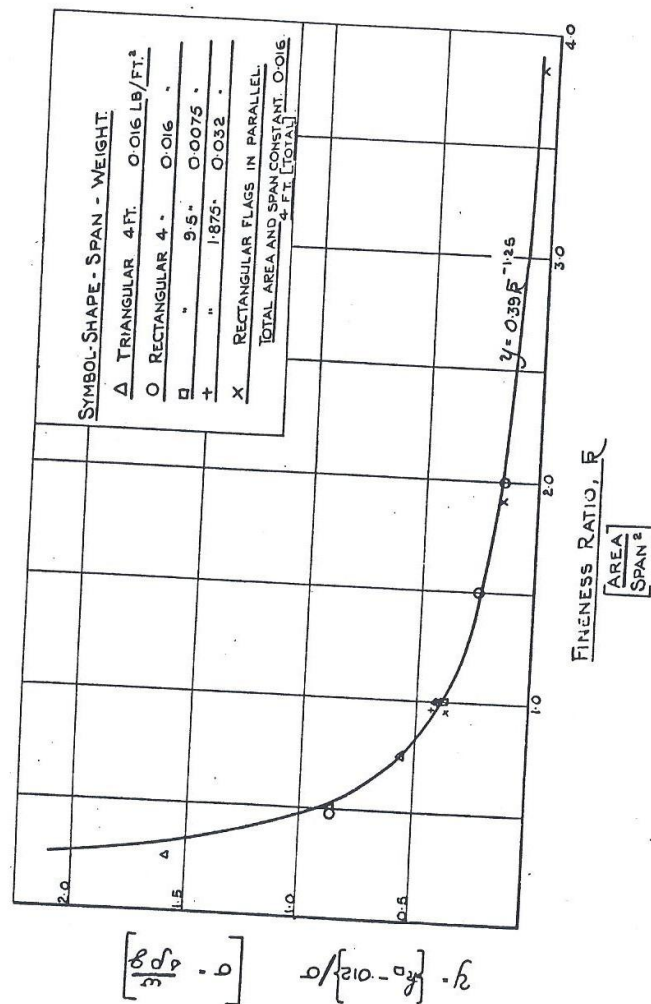
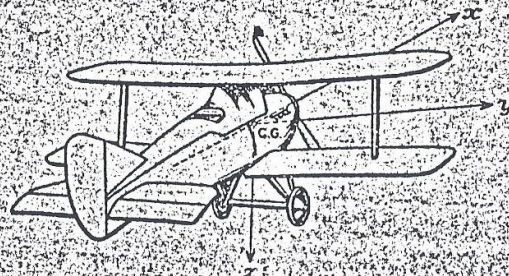


FIG. 2

SYSTEM OF AXES



Axes	Symbol Designation Positive direction	x longitudinal Forward	y lateral starboard	z normal downward
Force	Symbol	X	Y	Z
Moment	Symbol Designation	L rolling	M pitching	N yawing
Angle of Rotation	Symbol	ϕ	θ	ψ
Velocity	Linear Angular	u p	v q	w r
Moment of Inertia		A	B	C

Components of linear velocity and force are positive in the positive direction of the corresponding axis.

Components of angular velocity and moment are positive in the cyclic order y to z about the axis of x ; z to x about the axis of y ; and x to y about the axis of z .

The angular movement of a control surface (elevator or rudder) is governed by the same convention, the elevator angle being positive downwards and the rudder angle positive to port. The aileron angle is positive when the starboard aileron is down and the port aileron is up. A positive control angle normally gives rise to a negative moment about the corresponding axis. The symbols for the control angles are -

- δ aileron angle
- η elevator angle
- γ_r tail setting angle
- δ_r rudder angle